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Tungsten Lamp as Radiation Standard and the Emissivity Effects

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Abstract

The derivative of the detected incidance in a wavelength interval with respect to temperature includes two terms. The first term depends on the change in blackbody emission and the second one on the change of emissivity with temperature. The error of neglecting the second term is analyzed and evaluated for a standard radiation source, a tungsten lamp. In this case, the error changes form a negligible amount of 6% to a significant value of more than 45%.

1. Change of detected non-blackbody incidance with temperature

The temperature dependence of the detected incidance for thermal source in a wavelength interval has been evaluated for an ideal quantum detector and thermal detector.[1-4] In both cases, the temperature dependence of emissivity is a contributing factor. Thus, the derivative of the detected incidance in a wavelength interval with respect to temperature includes two terms: the first one is associated with the change in blackbody emission and the second one with the change of emissivity with temperature. The error of neglecting the latter is analyzed and evaluated for a specific source, a tungsten lamp, and a standard source in spectroscopy or thermal and vision problems.[5-8]

We are interested in finding how the detected incidance in a wavelength interval changes with temperature. We take a derivative of detected incidance with respect to temperature.

$$\frac{\partial\{E^{D}_{[\lambda 1,\lambda 2]}(T)\}}{\partial T} = \frac{\partial}{\partial T}\left\{\int_{\lambda_1}^{\lambda_2} \varepsilon(T,\lambda)\cdot R(T_D,\lambda)\cdot E_{\lambda}{}^{BB}(T)\cdot d\lambda\right\} \qquad [V/(m^2K)] \tag{1}$$

Even though emissivity changes little with either wavelength or temperature, the temperature effects are significant because increments in temperature are orders of magnitude larger than increments in wavelength (order of 6).[9]

$$\begin{aligned} E^{D}_{[\lambda 1,\lambda 2]T}(T) &= \varepsilon_T(T,\bar{\lambda})\cdot\int_{\lambda_1}^{\lambda_2} R(T_D,\lambda)\cdot E_{\lambda}{}^{BB}(T)\cdot d\lambda \\ &+ \varepsilon(T,\bar{\lambda})\cdot\int_{\lambda_1}^{\lambda_2} R(T_D,\lambda)\cdot E_{\lambda T}{}^{BB}(T)\cdot d\lambda \end{aligned} \qquad [V/(m^2K)] \tag{2}$$

1.1 Temperature dependence of the incidance detected with an ideal quantum detector

The incidance detected with a quantum detector depends on temperature. After we substitute the detector parameters for the quantum detector into Eq. (1), we may place the constants outside the integral.

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$$E^{D}_{[\lambda1,\lambda2]T}(T) = \frac{V_n}{\sqrt{A_D \cdot \Delta f}} \frac{D^*_2}{\lambda_2} \left[\begin{array}{l} \varepsilon_T(T;\bar{\lambda}) \int_{\lambda_1}^{\lambda_2} E_\lambda{}^{BB}(T)\lambda d\lambda + \\ \varepsilon(T;\bar{\lambda}) \int_{\lambda_1}^{\lambda_2} E_{\lambda T}{}^{BB}(T)\lambda d\lambda \end{array} \right] \quad [V/(m^2K)] \quad (3)$$

We substitute the Planck equation to obtain the spectral dependence of the incidance in the wavelength interval. After the evaluation of the integrals (see Refs. 1 and 3), and some further simplifications, Eq. (3) may be expressed as an infinite sum.[9]

$$E^{Dn}_{[\lambda1,\lambda2]T}(T) = \frac{c_1}{\lambda^2 \cdot \lambda_2} \sum_{m=1}^{\infty} \exp\left(\frac{-m \cdot c_2}{\lambda \cdot T} \right) \cdot \left\{ \frac{\varepsilon(T;\bar{\lambda})}{\lambda \cdot T} + \right. \quad [W/(m^2srK] \quad (4)$$

$$\left. + \frac{T \cdot \varepsilon_T(T;\bar{\lambda}) + 3 \cdot \varepsilon(T;\bar{\lambda})}{m \cdot c_2} \left(1 + 2\frac{T \cdot \lambda}{m \cdot c_2} + 2\frac{(T \cdot \lambda)^2}{(m \cdot c_2)^2} \right) \right\} \Bigg|_{\lambda=\lambda_1}^{\lambda_2}$$

The temperature dependence for the detected incidance from a thermal source in a wavelength interval for a quantum detector consists of two terms. The first one depends on the change in the blackbody emission with temperature. The second one incorporates the change of emissivity with temperature. Figure 1a shows the (normalized) temperature dependence of the incidance from a tungsten source detected with an ideal quantum detector.

<u>1.2 Temperature dependence of the incidance detected with an ideal thermal detector</u>

Now we are interested in calculating how the incidance from a thermal source detected with a thermal detector depends on temperature. We follow the same steps as above to obtain equivalent results.

$$E^{Dn}_{[\lambda1,\lambda2]T}(T) = \left\{ \frac{c_1}{\lambda^3} \sum_{m=1}^{\infty} \exp\left(\frac{-m \cdot c_2}{\lambda \cdot T} \right) \cdot \left\{ \frac{\varepsilon(T;\bar{\lambda})}{\lambda \cdot T} + \right. \right. \quad [W/(m^2srK] \quad (5)$$

$$\left. \left. + \frac{T\varepsilon_T(T;\bar{\lambda}) + 4\varepsilon(T;\bar{\lambda})}{mc_2} \left(1 + 3\frac{T\lambda}{mc_2} + 6\frac{(T\lambda)^2}{(mc_2)^2} + 6\frac{(T\lambda)^3}{(mc_2)^3} \right) \right\} \cdot \Bigg|_{\lambda=\lambda_1}^{\lambda_2} \right\} .$$

Equation (5) gives the temperature dependence for the (normalized) detected incidance from a thermal source in a wavelength interval for a thermal detector. Figure 1b shows the (normalized) temperature dependence for the incidance from a tungsten source detected with an ideal thermal detector.

2. Emissivity and its change with temperature: analysis

Temperature dependence of the expression for the detected incidance of thermal radiation in a wavelength interval has a factor F^A multiplying the emissivity and factor F^B multiplying the emissivity change with temperature.

$$F^{A}_{[\lambda1,\lambda2]}(T) = \int_{\lambda_1}^{\lambda_2} R(T_D,\lambda) \cdot E_{\lambda T}{}^{BB}(T) \cdot d\lambda \quad (6)$$

$$F^{B}_{[\lambda1,\lambda2]}(T) = \int_{\lambda_1}^{\lambda_2} R(T_D,\lambda) \cdot E_{\lambda}{}^{BB}(T) \cdot d\lambda \quad (7)$$

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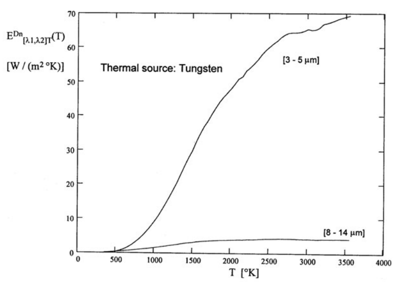
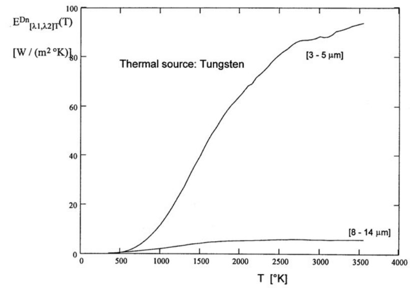


(a) ideal quantum detector



(b) ideal thermal detector

Fig. 1. The (normalized) temperature dependence of the detected incidance from a tungsten source in two wavelength intervals, incorporating measured emissivity data.

The historically accepted assumption has been that the change of emissivity with temperature is so small that the term incorporating it can be ignored. As an example of measured data, Figure 2a shows the emissivity of the tungsten as a function of temperature, while Fig. 2b exhibits its temperature derivative as a function of temperature. Its small values explain the reasons for neglecting its contributions in the past. However, its small value does not take into account the magnitude of factor F^A/F^B. Their product results sufficiently large to question the historical assumption.

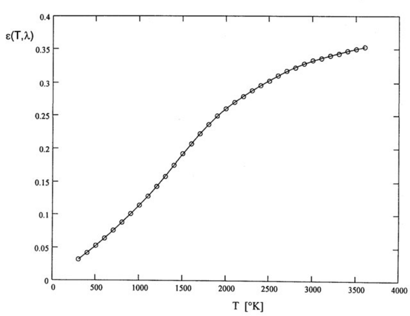


Fig. 2a. Tungsten emissivity as a function of temperature (after Weast [10]).

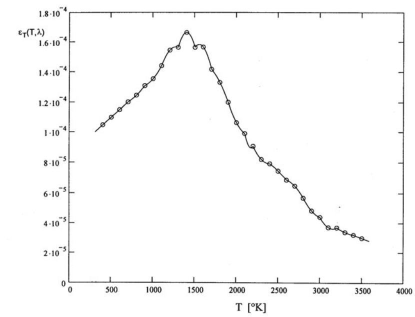


Fig. 2b. Derivative of tungsten emissivity with respect to temperature as a function of temperature.

3. Relative error when neglecting change of emissivity with temperature

The relative error $e_R(T)$ of ignoring the term incorporating the emissivity change with temperature may be found.

$$e_R(T) = \frac{\varepsilon_T(T,\bar{\lambda})\cdot\int_{\lambda_1}^{\lambda_2} R(T_D,\lambda)\cdot E_\lambda{}^{BB}(T)\cdot d\lambda}{\left[\begin{array}{l}\varepsilon_T(T,\bar{\lambda})\int_{\lambda_1}^{\lambda_2} R(T_D,\lambda)E_\lambda{}^{BB}(T)d\lambda \\ +\,\varepsilon(T,\bar{\lambda})\int_{\lambda_1}^{\lambda_2} R(T_D,\lambda)E_{\lambda T}{}^{BB}(T)d\lambda\end{array}\right]} \qquad (8)$$

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We may rewrite the relative error in a more compact form, as 1/(1+r).

$$e_R(T) = \frac{1}{1 + \dfrac{\varepsilon(T,\bar{\lambda})F^A_{[\lambda 1,\lambda 2]}(T)}{\varepsilon_T(T,\bar{\lambda})F^B_{[\lambda 1,\lambda 2]}(T)}} \qquad (9)$$

Figure 3a shows graph of the fraction, 1/(1+r). For value of *r* greater than 99, the relative error is less than 1%. Figure 3b shows the relative error as a function of the quotient F^A/F^B. Ratio of the emissivity to its derivative with temperature ($\varepsilon/\varepsilon_T$) is a parameter. Figure 3c shows the ratio of the emissivity to its change with temperature ($\varepsilon/\varepsilon_T$) for a tungsten source as a function of the temperature.

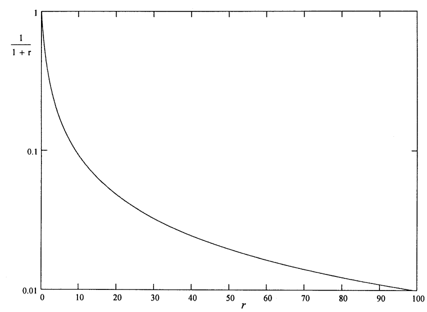

Fig. 3a. Relative error as a function of factors represented by r. Note the logarithmic scale.

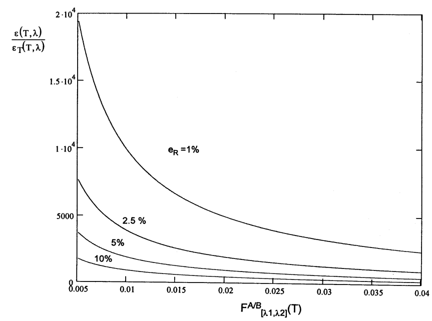

Fig. 3b. Relative error as a function of the quotient F^A/F^B. Ratio of the emissivity to its derivative with temperature ($\varepsilon/\varepsilon_T$) is a parameter.

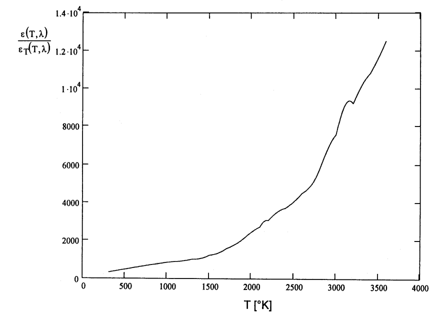

Fig. 3c. Ratio of the emissivity to its change with temperature ($\varepsilon/\varepsilon_T$) for a tungsten source increases rapidly with temperature.

The quotient F^A/F^B must be greater than 0.00857 for the tungsten thermal source to maintain a relative error of less than 1%. For a tungsten source, we compare its emissivity to its change of emissivity with temperature. At 3500 °K the tungsten has emissivity of 0.351 and the change of emissivity with temperature of $3\text{x}10^{-5}$ °K^{-1}. This number is not easily controlled, because it depends on the wavelength interval, temperature, and the detector employed.

3.1 Relative error for a quantum detector

Figure 4a shows the relative error arising when the change of emissivity with temperature is neglected, for a tungsten source and for two wavelength intervals. The error increases from 6% to 45%. Next, we compare F^A and F^B in a wavelength interval of common use (for example, [3 to 5 μm] and [8 to 14μm]). Figure 4b shows the quotient F^A/F^B of the temperature dependence of the incidance in wavelength interval detected with an ideal quantum detector.

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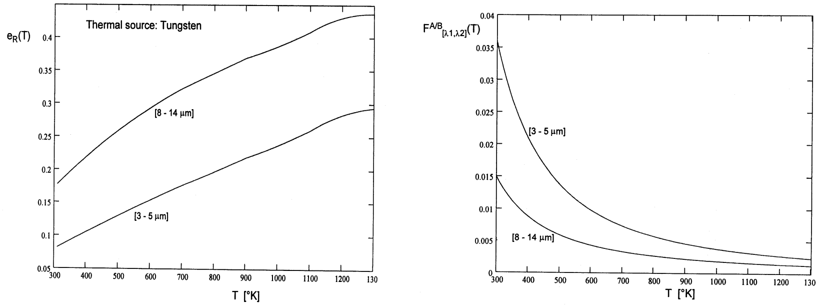


(a) error of neglecting emissivity (b) quotient F^A/F^B

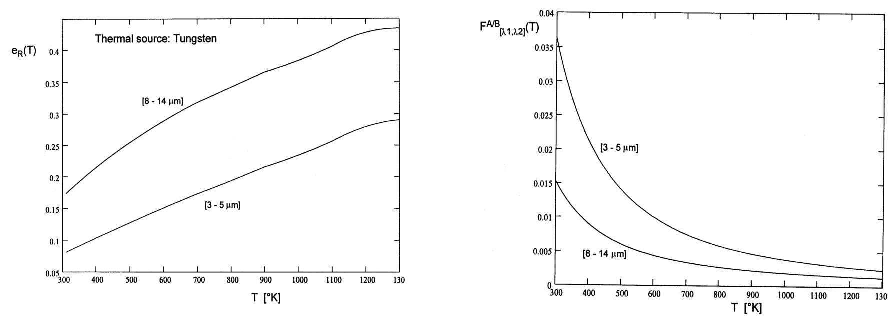
Fig. 4. The error of neglecting the temperature dependence of emissivity (a) and the quotient F^A/F^B (b) as a function of temperature in two wavelength intervals, detected with an ideal quantum detector.

3.2. Relative error for a thermal detector

Figure 5a shows the relative error arising when the change of emissivity with temperature is neglected for a tungsten source, in two wavelength intervals. The values form 6% to more than 45% are significant. Figure 5b shows the quotient F^A/F^B in the temperature dependence of the incidance in two wavelength intervals detected with an ideal thermal detector.

4. Conclusions

Our initial objective was to determine whether a small change in temperature and/or emissivity might be used to monitor the state of health of an agricultural crop, or to help us detect any faint changes. We developed analytical expressions for the derivative of the incidance from thermal radiator detected with both, thermal and quantum detectors. We implemented their solutions as infinite series and displayed results graphically.



(a) error of neglecting emissivity (b) quotient F^A/F^B

Fig. 5. The error of neglecting the temperature dependence of emissivity (a) and the quotient F^A/F^B (b) as a function of temperature in two wavelength intervals, detected with an ideal thermal detector.

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The expression for the change of incidance detected with a quantum detector indeed consists of two terms, one with emissivity and the other with the derivative of emissivity multiplied by temperature. Thus, the small emissivity change is multiplied by a factor of 300 for room temperature applications. We note that the constant term in each case contains only the emissivity divided by the product of wavelength and temperature.

We used published emissivity data for such common thermal radiator as a tungsten source, and a radiometric standard in spectrometry. We found that the error of neglecting the term with emissivity change ranges from 6% at room temperature to nearly 50% at 1300 K, not quite the operational temperature for this device.

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